COLLEGE PREPARATORY MATHEMATICS

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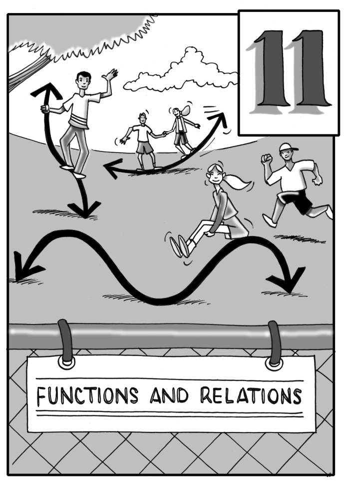
Fun Functions: Making Functions Active and Interesting



For more information about the materials you find in this packet, contact:

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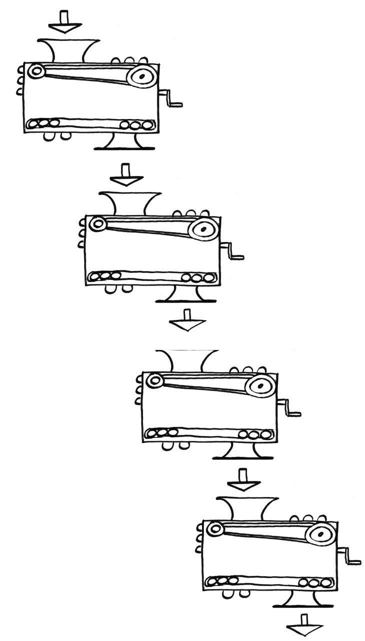
1-2. FUNCTION MACHINES

Your teacher will give you a set of four function machines. Your team's job is to get a specific output by putting those machines in a particular order so that one machine's output becomes the next machine's input. As you work, discuss what you know about the kind of output each function produces to help you arrange the machines in an appropriate order. The four functions are reprinted below.

$f(x) = \sqrt{x}$ $\quad\quad$ $g(x) = -(x-2)^2$

$h(x) = 2^x - 7$ $\quad\quad$ $k(x) = -\frac{x}{2} - 1$



a. In what order should you stack the machines so that when 6 is dropped into the first machine, and all four machines have had their effect, the last machine's output is 11?

b. What order will result in a final output of 131,065 when the first input is 64?

URL for Function Machines:

http://www.cpm.org/pdfs/stuRes/CCA2/chapter_01/CCA2%20Lesson%201.1.1%20RP.pdf

3-9. SILENT BOARD GAME (SBG)

During Lesson 3.1.1, you created written rules for patterns that had no tiles or numbers. You will now write algebraic rules using a table of jumbled in/out numbers. Focus on finding patterns and writing rules as you play the Silent Board Game. Your teacher will put an incomplete $x \rightarrow y$ table on the board. Study the input and output values and look for a pattern. Then write the rule in words and symbols that finds each y-value from its x-value.

URL for SBG
http://www.cpm.org/pdfs/stuRes/CC3/chapter_03/CC3%20Lesson%203.1.2A%20RP.pdf

ALGEBRA/Function WALK

Materials Preparation: Mark intervals on each rope by tying a ribbon into a knot every 30". Number each set of index cards with integers from –6 to 6. Use colors that coordinate with the sticky dot colors.

Create and label axes on the poster paper with values from –6 to 6 (or –10 to 10). Color the axes to match the color of the sticky dots and title with the corresponding equation in color. Put an IN (x) → OUT (y) table like the one shown in problem 1-17 at the bottom of each poster. Optionally, laminate the poster paper. Have the poster graphs up and ready in the classroom for students to plot their (x, y) points after returning from the Algebra Walk.

Lesson Overview: Today is the Algebra Walk, from the MCTP Professional Development Package, a compilation of classroom-tested ideas from Australian mathematics teachers. This activity incorporates most of the elements of an *xy*-coordinate system. Students role play "human points" that they associate with their given coordinates. They also simulate movement of points on an *xy*-coordinate system. This kinesthetic experience will help students understand the role of coordinates, why they form ordered pairs, the rules for moving from the origin to a specific point on the grid, and very informally experience the effect that the constant and coefficient have on a linear graph in the slope-intercept form, $y = mx + b$.

ALGEBRA WALK PROCEDURE:

1. Before going outside, give each student a data sheet (Lesson 1.1.3 Resource Page- http://www.cpm.org/pdfs/stuRes/CC3/chapter_01/CC3%20Lesson%201.1.3A%20RP.pdf) and at least one index card with one of the integers between –6 and 6, written on the left half of the card. There should be thirteen cards for each color, one for each integer x-value from –6 to 6. Each color relates to one of the equations in parts (a) through (e) of problem 1–17. Each student will also need a pencil for sketching the "human graphs" while outdoors. There should be enough cards for students to have more than one card. Make sure that each student has cards that are different in color and remind students NOT to write on the cards.

2. Once outside, situate students so they are facing the x-axis, looking toward the positive y-direction. This orientation is important because it corresponds to the standard orientation used when graphing.

   Call for students with red cards to find their places along the horizontal axis. The students should stand with both feet on the x-axis facing the positive "y" direction, with their backs to the rest of the class. Start with part (a) of problem 1-17, $y = 2x + 1$. Give the following directions: "*Be sure you are standing on the mark that corresponds to the number on your card. Look at the number. Multiply it by 2. Add one.* (Pause) *Got it? Record the resulting number on your resource page. When I say 'go,' take that number of paces forward or backward, parallel to the y-axis. A 'pace' is the distance between two marks on the vertical axis. Ready? GO!*"

   Mistakes will be made. Encourage students to help each other out. In most cases, the students will handle corrections themselves. Resist the urge to manage this yourself.

3. Have the student observers complete the appropriate section of their data sheet. They should roughly sketch and describe each shape they see. You may need to modify the sheet if the colors of your cards and dots are different.
4. Repeat this process for each rule on the resource page. You might have students who do rule (c) stay in position while others graph rule (d), to introduce the idea of the intersection of lines.

   If you want to extend the exercise, have a set of students take two steps to their right after they have created a graph of a function. Ask them what features of the graph change and what features stay the same. This begins an intuitive introduction to translations that will appear occasionally during the year.
5. Back in class: Record data from each function on one large graph using poster graph paper and sticky dots. Have teams of students record their (x, y) coordinates in tables for each rule on the chalkboard, or ask for verbal responses for each separate graph and record the data yourself.

Hint: Have a piece of yarn for the students to hold as they make a line. This will help them to see that the line is continuous and not discrete. This is especially useful in the Function Walk.

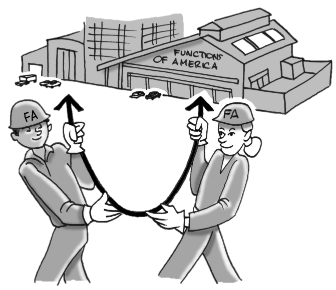
If the outdoor activity cannot be done due to weather, one alternative is to do the activity as described using the floor of your classroom. Another method is to use large poster graph paper, but this should be avoided if at all possible. Doing this problem outside makes this one of the most memorable and enjoyable problems of the year.

FUNCTION WALK: To extend this to a Function Walk ask the second or third group that comes up, what the Domain is to start. If they are holding the yarn, remind them that it goes on past the endpoints so that the Domain is all reals. Then, after they make their line, have the students face the y-axis and walk to it. The Range will be all the points that are covered by the yarn. You can use equations with asymptotes and add that into the discussion. y=1/x is a good one with which to end.

1.1.3 What do I know about a parabola?

Investigating the Graphs of Quadratic Functions

In the previous lesson, you observed linear, inverse variation, and exponential functions. In this lesson, you will study equations that create a family of functions called **quadratics**. The graph of a quadratic function has the shape of a **parabola**. You will learn all you can about their shape.



1-23. FUNCTIONS OF AMERICA

Congratulations! You have just been hired to work at a national corporation called Functions of America. Recently your company has had some growing pains, and your new boss has turned to your team for help. See her memo below.

MEMO

To: Your study team
From: Ms. Freda Function, CEO
Re: New product line

I have heard that while lines are very popular, there is a new craze in Europe to have non-linear designs. I recently visited Paris and Milan and discovered that we are behind the times!

Please investigate a new function called a quadratic function. A quadratic function can be written in the form $y = ax^2 + bx + c$*. Quadratic functions have the shape of a parabola.*

I'd like a full report at the end of today with any information your team can give me about its shape and equation. Spare no detail! I'd like to know everything you can tell me about how the equation for a quadratic function affects its shape. I'd also like to know about any special points on a parabola or any patterns that exist in its table.

Remember, the company is only as good as its employees! I need you to uncover the secrets that our competitors do not know.

Sincerely,
Ms. Function, CEO

Problem continues on next page. →

1-23. *Problem continued from previous page.*

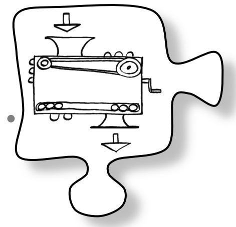
Your Task: Your team will be assigned its own quadratic function to study. Investigate your team's function and be ready to describe everything you can about it by using its graph (which is in the shape of a parabola), equation, and table. Answer the questions below to get your investigation started. You may answer them in any order; however, do not limit yourselves to these questions!

- How would you describe the shape of your parabola? For example, would you describe your parabola as opening up or down? Do the sides of the parabola ever go straight up or down (vertically)? Why or why not? Is there anything else special about its shape?
- Does your parabola have any **lines of symmetry**? That is, can you fold the graph of your parabola so that each side of the fold exactly matches the other? If so, where would the fold be? Do you think this works for all parabolas? Why or why not? For more information on lines of symmetry, see the Math Notes box at the end of this lesson.
- Are there any special points on your parabola? Which points do you think are important to know?
- Are there *x*- and *y*-intercepts? What are they? Are there any intercepts that you expected but do not exist for your parabola?
- Is there a highest (maximum) or lowest (minimum) point on the graph of your parabola? If so, where is it? This point is called a **vertex**.

List of Quadratic Functions:

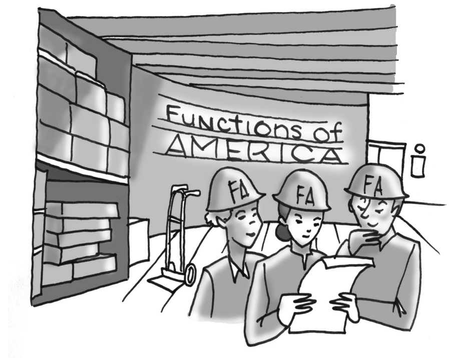
$y=x^2-2x-8$	$y=-x^2+4$
$y=x^2-4x+5$	$y=x^2-2x+1$
$y=x^2-6x+5$	$y=-x^2+3x+4$
$y=-x^2+2x-1$	$y=x^2+5x+1$

11.1.1 How can I make it different?



Transforming Functions

11-1. FUNCTIONS OF AMERICA



Your customers at Functions of America want unique lines and curves—not the same ones used by everybody else. However, they cannot afford custom-made functions. Instead you suggest that they order small modifications to in-stock functions. Perhaps your customers would be happy adding (or subtracting) a little bit to $f(x)=2x+4$ or $f(x)=3x^2-4$.

a. Consider the transformation $f(x)+k$ where k is any real number. What is the impact of this transformation on the original line, $f(x)=2x+4$? Investigate the impact on both the graph of the line and the algebraic equation of the line.

b. What is the impact of the transformation $f(x)+k$ on a parabola like $f(x)=3x^2-4$? Again, investigate the impact on both the graph of the parabola and its algebraic equation.

11-2. Investigate the impact of transforming a function by adding or multiplying a constant k, where k is any rational number. Consider the following transformations:

$f(x)+k$ $\qquad$ $f(kx)$ $\qquad$ $k\cdot f(x)$ $\qquad$ $f(x+k)$

Your Task: Your teacher will assign your team one or more of the functions below.

a. $f(x)=-3x+2$ $\qquad$ b. $f(x)=x^2-6$

c. $f(x)=1.5\cdot 3^x$ $\qquad$ d. $f(x)=-2x^2+1$

e. $f(x)=4x-4$ $\qquad$ f. $f(x)=5\cdot(\frac{1}{2})^x$

g. $f(x)=-1.5x+4$ $\qquad$ h. $f(x)=\frac{1}{2}x^2-3$

11-2. *Problem continued from previous page.*

Prepare a poster presentation with your conclusions. Include enough information on your poster so that the impact of k is very clearly communicated without further explanation. Consider both the graph and the algebraic equations.

As you prepare your poster, consider the following discussion points.

Discussion Points

What impact does k have on the growth of the function?

How does k change the starting point or vertex?

How do changes in the graph connect to changes in the equation?

Does k slide the graph up or down? Left or right?

What if k is negative?

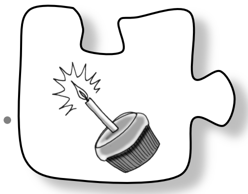
11-94. FUNKY FUNCTION

The following relation combines two functions you have studied to make a new kind of graph.

$$f(x)=2\left|3-2x-x^2\right|+4$$

With your team, create a *complete* description of the graph of $f(x)$. Find (or verify) coordinates of special points by evaluating the equation.

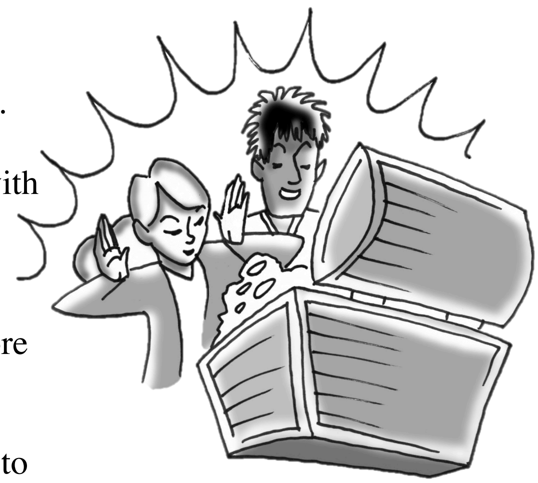
11.3.2 Can I find it?



Relation Treasure Hunt

Relations include mathematical relationships between two variables that are functions, but also relationships that are not functions. Today you will use your knowledge to distinguish between different relations that are given in different representations.

11-86. TREASURE HUNT



Today your teacher will give you several descriptive clues about different relations. (This information is also available online at www.cpm.org.) For each clue, work with your team (or a partner) to find all the possible matches among the relations posted around the classroom or provided on the resource page. Remember that more than one relation may match each clue. Once you have decided which relation(s) match a given clue, defend your decision to your teacher and receive the next clue. Be sure to record your matches on paper.

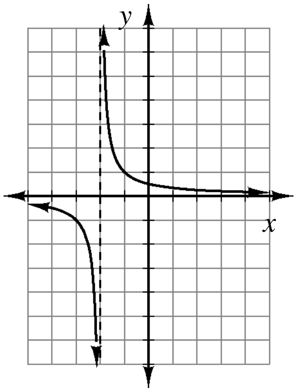
Your goal is to find the match (or more than one match) for each of eight clues. Once you and your team (or partner) have finished, only one relation will be left unmatched. That relation is the treasure!

URL for Relation cards and clues:
http://www.cpm.org/pdfs/stuRes/CCA/chapter_11/CCA%20Lesson%2011.3.2A%20RP.pdf
http://www.cpm.org/pdfs/stuRes/CCA/chapter_11/CCA%20Lesson%2011.3.2B%20RP.pdf
http://www.cpm.org/pdfs/stuRes/CCA/chapter_11/CCA%20Lesson%2011.3.2C%20RP.pdf
http://www.cpm.org/pdfs/stuRes/CCA/chapter_11/CCA%20Lesson%2011.3.2D%20RP.pdf

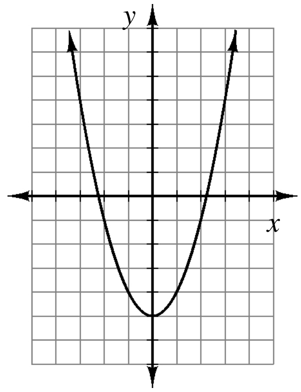
CAROUSEL

2-95. Write a possible equation for each of these graphs. Assume that one mark on each axis is one unit. When you are in class, check your equations on a graphing calculator and compare your results with your teammates.

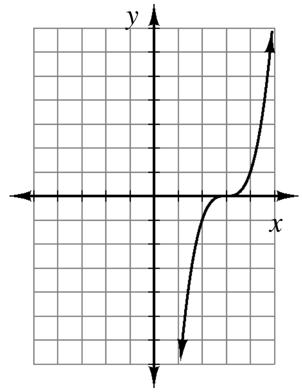
a.



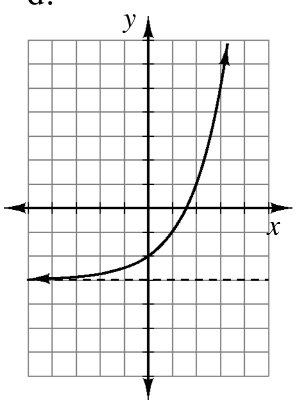
b.



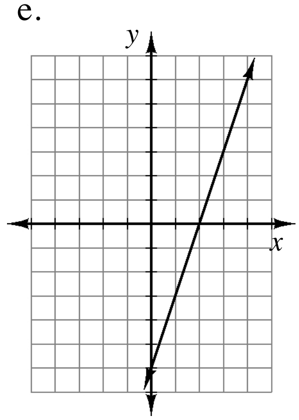
c.



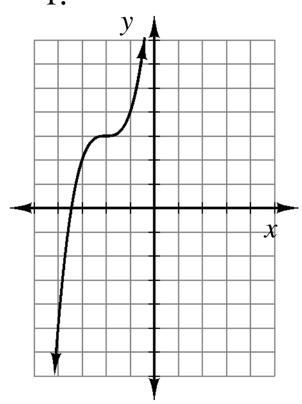
d.



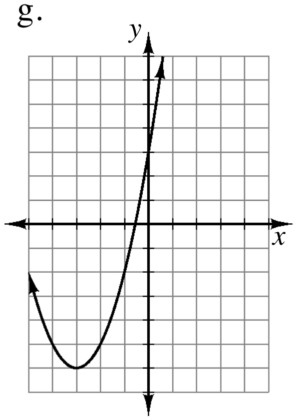
e.



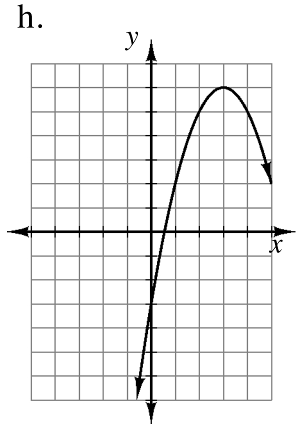
f.



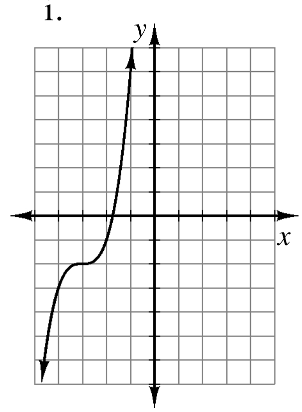
g.



h.



i.



Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

1 Make sense of problems and persevere in solving them.

- **Find meaning in problems**
- **Look for entry points**
- **Analyze, conjecture and plan solution pathways**
- **Monitor and adjust**
- **Verify answers**
- **Ask themselves the question: “Does this make sense?”**

2 Reason abstractly and quantitatively.

- **Make sense of quantities and their relationships in problems**
- **Learn to contextualize and decontextualize**
- **Create coherent representations of problems**

3 Construct viable arguments and critique the reasoning of others.

- **Understand and use information to construct arguments**
- **Make and explore the truth of conjectures**
- **Recognize and use counterexamples**
- **Justify conclusions and respond to arguments of others**

4 Model with mathematics.

- **Apply mathematics to problems in everyday life**
- **Make assumptions and approximations to simplify a complicated situation**
- **Identify quantities in a practical situation**
- **Interpret results in the context of the situation and reflect on whether the results make sense**

5 Use appropriate tools strategically.

- **Consider the available tools when solving problems**
- **Are familiar with tools appropriate for their grade or course (pencil and paper, concrete models, ruler, protractor, calculator, spreadsheet, computer programs, digital content located on a website, and other technological tools)**
- **Make sound decisions of which of these tools might be helpful**

6 Attend to precision.

- **Communicate precisely to others**
- **Use clear definitions, state the meaning of symbols and are careful about specifying units of measure and labeling axes**
- **Calculate accurately and efficiently**

7 Look for and make use of structure.

- **Discern patterns and structures**
- **Can step back for an overview and shift perspective**
- **See complicated things as single objects or as being composed of several objects**

8 Look for and express regularity in repeated reasoning.

- **Notice if calculations are repeated and look both for general methods and shortcuts**
- **In solving problems, maintain oversight of the process while attending to detail**
- **Evaluate the reasonableness of their immediate results**